

Non-linear system of simultaneous equations

1. Solve for real numbers x, y :

$$\begin{cases} 4x^2 + y^2 = 17 + 4x \\ (2x - 1)^2 + (y - 8)^2 = 34 \end{cases}$$

2. Solve for real numbers:

$$\begin{cases} w + x + y + z = 10 \\ w^2 + x^2 + y^2 + z^2 = 30 \\ w^3 + x^3 + y^3 + z^3 = 100 \\ wxyz = 24 \end{cases}$$

Hint : Inspection.

3. Given:

$$\begin{cases} a^2 + b^2 + ab = 9 \\ b^2 + c^2 + bc = 16 \\ c^2 + a^2 + ca = 25 \end{cases}$$

(a) If $a, b, c > 0$, find $ab + bc + ca$.

(b) **(Hard)** If a, b, c are real numbers, find $ab + bc + ca$.

Hint : Converse of Pythagoras Theorem, Cosine Law and Area of Triangle.

4. **(Hard)** Solve for real numbers a, b, c :

$$\begin{cases} a^2 - 2ab + bc - c^2 + ca = 0 \\ b^2 - 2bc + ca - a^2 + ab = 0 \\ c^2 - 2ca + ab - b^2 + bc = 0 \end{cases}$$

Hint: The system is cyclic, without loss of generality, let $a \geq b \geq c$.